The Reticular Action Model: A Remarkably Lasting Achievement

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Abstract

This chapter begins with a brief history of path analysis and structural equation modeling leading up to John J. McArdle's development of the Reticular Action Model (RAM). The path analysis work of Sewall Wright and the reticule ideas of Raymond Cattell were seminal to how the RAM model was developed. The history is meant to evoke a sense of how surprising the development of RAM was given what was known up to that time. Next is an overview of how MacDonald and McArdle solved the infinite sum problem introduced by digraphs with cycles; what was at the time called a non-recursive model. The chapter then takes a more personal turn as I recollect some conversations with McArdle in the 1980s when the path diagrammatic constituents of RAM graphs were developed. The chapter concludes with discussion of how thinking about Structural Equation Models has evolved over the past 40 years and how two generations of PhD students learning the RAM way of thinking has influenced that discussion.

For readers whose first experience of Structural Equation Modeling (SEM) occurred within the past 40 years, the Reticular Action Model (RAM) approach to SEM may seem obvious. However, prior to McArdle (1978) first presenting his formulation, this way of approaching data analysis differed from contemporary approaches in fundamental ways. I will approach this chapter with a more personal perspective than is the norm. The reason for this is that I was living in Denver at the time of RAM's development and had the

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good fortune to be friends with Jack McArdle while he was a postdoc with John Horn. Jack and I met most Thursdays for lunch at a Mexican restaurant or in his basement office where he used me as a sounding board for the arguments, algebra, and diagrams that underlie the RAM model. These discussions were of equal parts matrix algebra and graph theory; defining characteristics of the RAM way of specifying models. The current chapter will begin with some historical context, give an exposition of why the RAM model was a breakthrough, and then trace some of the developments that RAM has enabled.

Setting the Stage

Tukey wrote, "The important question about methods is not 'How' but 'Why.' ... In explaining why, we must remember that every method is to some extent the child of the time of its development." (Tukey, 1954, p. 33). With this in mind, I will begin with a short history of data analysis from a path analytic perspective to provide historical context for the shift in thinking that led to the development of the RAM model and the surprising capabilities that it made available to modelers (see Li et al., 1975; Wolfle, 1999, 2003, for further historical accounts of path analysis).

Karl Pearson (1896) developed the *correlation coefficient* which is, at its heart, an average cross product between two mean centered variables then rescaled to be in units of the product of the root mean squares of the same variables. Keep in mind that the cross products and sums of squares (or covariance and variance) are essentially the same operation on either two variables or one variable respectively.

From this idea of averaging sums of squares as a measure of the relatedness of variables, Spearman (1904) built his method of *factor analysis*. The method can be framed such that an unobserved variable would be the common cause for a number of observed variables and could be estimated as a system of simultaneous equations with unobserved common factor(s) on the right hand side of each equation and one equation for each observed variable.

Sewall Wright (1918) derived *path coefficients* that decomposed a single observed correlation into multiple parts and thereby demonstrated a general size factor as well as leg and skull size factors in rabbits. Two years later, he wrote, "The correlation between two variables can be shown to equal the sum of the products of the chains of path coefficients along all of the paths by which they are connected." (Wright, 1920, p 330) and used this property to demonstrate heritable traits in piebald guinea pigs. Wright also worked out what we now call the *components of correlation.*

> It can be shown that the squares of the path coefficients measure the degree of determination by each cause. If the causes are independent of each other, the sum of the squared path coefficients is unity. If the causes are correlated, terms representing joint determination must be recognized. The complete determination of X [...] by factor A and the correlated factors B and C, can be expressed by the equation: $a^2 + b^2 + c^2 + 2bcr_{BC} = 1$ (Wright, 1920, p 329)

Wright's seminal articles not only established the basis of structural equation modeling, but also provided the first examples of path diagrams (see Figure 1). Wright used his path diagrams to explain the implications of his equations, thus demonstrating that these path diagrams were more than just a convenience, but had a fundamental relationship to the equations (see McArdle & Aber, 1990, for an SEM analysis of Wright's data). In an article published the next year, Wright (1921) again combined use of the path diagrams and simultaneous equations to illustrate a variety of structural models. He makes the important point that one should not interpret these equations as being statements of causality, but only as being structures of correlations. It is beguiling to think that the left hand side of the simultaneous equations of SEM are outcomes of the right hand side. But the equals sign is not the same as a "stores into" operator, equality only expresses a network of balances of the left and right hand sides. This balancing of accounts is at the heart of SEM, but some model formulations encourage causal interpretation more than others (Pearl, 2003).

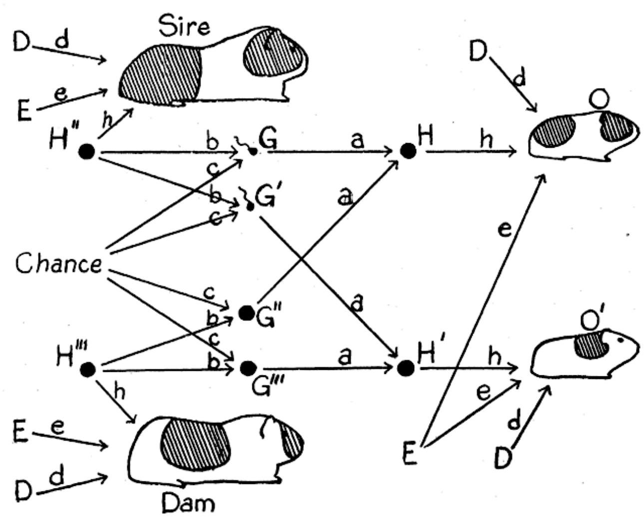

Figure 1. The first published example of a path diagram. Note modern features of SEM including the exogenous variable labeled "Chance", common environment (E), unique environment (D) and additive genetic paths (a) constrained to equality across generations (Wright, 1920).

Wright's path analysis method did not immediately become popular. Parameters of systems of simultaneous equations with constraints were difficult to estimate from data when "Computer" was not a noun applied to a machine, but rather was a job title for people who calculated arithmetic operations as a profession and used paper and pencil as tools. Wright took an approach that would now be called an *instrumental variables* approach, isolating parts of the model and estimating them independently (see, e.g., Bollen, 1996; Kirby & Bollen, 2009, for a modern use of this technique). In 1934, Wright proposed that the method of path coefficients could be converted into Pearson's partial regression coefficients, but it was another 20 years before the method began to attract attention.

Tukey gave an idea of how far path analysis had faded into the background when he wrote "... I had heard of path coefficients repeatedly. The surprise came when I found that I did not know anything about path analysis, although after some study it seemed to be natural and useful. After coming to the point where I thought I understood it moderately well, it occurred to me to wonder why I had not known about it before." (Tukey, 1954, p.35). Tukey (1954) and later Turner and Stevens (Turner & Stevens, 1959) proposed that unstandardized (what Turner and Stevens called "concrete") coefficients be used for path analysis and that reciprocal relations be allowed in path analytic models.

Wright's responses (1960a, 1960b) to Tukey brought the discussion of path analysis back into the literature. Wright mentions that overdetermined systems may be estimated if one can "... obtain a compromise solution by the method of least squares..." (Wright, 1960a, p. 198). At the time, the method used for obtaining solutions to overdetermined models was not simultaneous equations optimization, but instead isolating instrumental variables and then solving one equation at a time.

Cattell, steeped in the traditions of factor analysis, argued that the notion of factors arranging themselves in a pyramid with fewer factors at higher levels was an inevitable mathematical artifact and not a proof of hierarchical structure of psychological constructs (see Figure 2).

> "One mathematical rule, when communalities are used, is that one cannot take out as many factors as there are variables. Consequently, a hundred variables may define, say, only twenty primaries (*n.b.*, first order factors), and twenty primaries must yield fewer second order factors, and so on. But the fact that a number of higher order factors as great as the number of variables or lower order factors cannot be mathematically defined for lack of a sufficiency of variables is no proof that they do not exist." —R. B. Cattell (1965)

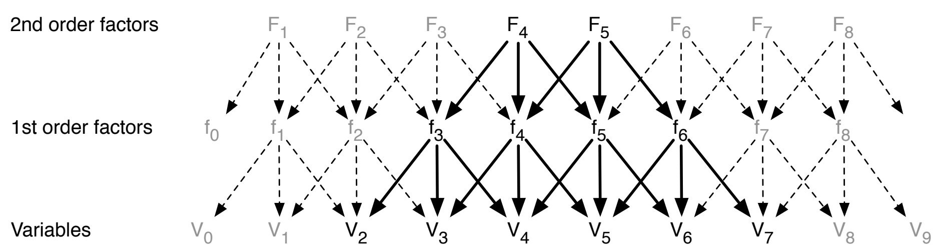

Figure 2. Alternative explanation for a pyramidal organization of higher order factors. If only variables V2 through V7 are measured, a hierarchical structure is inevitable. Figure adapted from Cattell (1965, p. 234) who captioned his figure, "False hierarchy in an essentially reticular structure."

In that chapter, Cattell argued for a *reticular* (i.e., network or directed graph) structure for latent variables as shown in Figure 3 adapted from his chapter. This model includes all possible reciprocal regression relations between variables and first order factors. Cattell states that he did not draw all the possible reciprocal relations from the second order factors in order "to avoid overcrowding".

After mainframe computers started to become commonplace in the 1950s and 1960s, improved techniques for optimization became available and interest was revived. Duncan (1966) reviewed Wright's articles in detail and brought path coefficients and path analysis to wider attention in the social sciences. Duncan recognized the value in path diagrams being an exact representation of the underlying algebra and complained "Causal diagrams are appearing with increasing frequency in sociological publications. Most often, these have some kind of pictorial or mnemonic function without being isomorphic with the algebraic

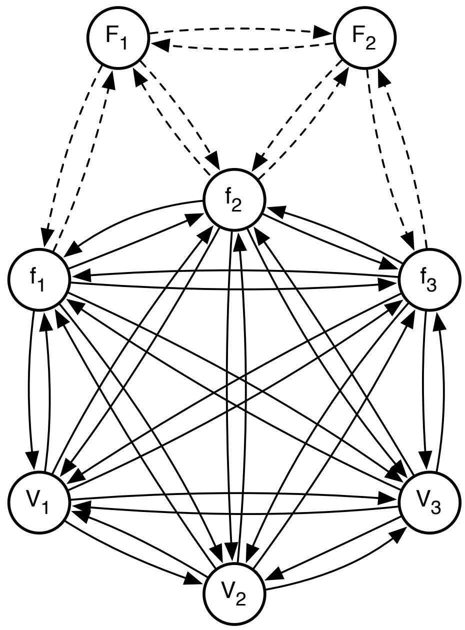

Figure 3. Cattell's reticular structure presages modern *network* or *graph* models. Figure is adapted from Cattell (1965, p. 236) who captioned his figure, "Possible interactions of a set of eight distinct influences."

and statistical properties of the postulated system of variables—or, indeed without having a counterpart in any clearly specified system of variables at all." (Duncan, 1966, p. 3). At this point, while the path diagrams of Wright provided a unified way to view a wide variety of models, fitting models to data involved algorithms specialized to particular models. A clearer unified view of linear structure of correlation or covariance was still in the future. The separate languages used in ANOVA, multiple regression, factor analysis, and path analyses also inhibited unification of the field.

Jöreskog (Jöreskog, 1970) first presented a unified matrix-oriented model for the relations between variables. Goldberger noted the relationship between Jöreskog's formulation and Wright's path coefficients models and published a review in order to "redress economists' neglect of the work of Sewall Wright." (Goldberger, 1972, p. 979). Goldberger organized a series of meetings between members of the psychometric (including Jöreskog), econometric, and sociological (including Duncan) communities. At the first of these, Jöreskog (Jöreskog, 1973) presented his Linear Structural Relations (LISREL) model. LISREL allowed the specification of a wide variety of statistical models within the framework of its matrices.

Each matrix in the LISREL system of specification corresponded to part of an SEM model. Once the matrices were specified, the same optimization procedure could be used to obtain parameter estimates from the model no matter which model was input into the matrices. This was an incredible improvement over the previously idiosyncratic methods for estimation and accelerated the adoption of SEM in the social and behavioral sciences. One property of the LISREL system is that it provides definitions of how the matrices should be used and thus structures the thinking of modelers. This has the benefit of providing instructional support for someone first learning how to create statistical models. But it also has the disadvantage of reducing the probability that someone will innovate new structural relations that are not easily described within the LISREL framework. It was not uncommon for researchers in the 1970s and 1980s to speak of "tricking LISREL" into fitting the model they had in mind.

Starting in the late 1960s (e.g., Cohen, 1968) and accelerating in the late 1970s, people in the variety of fields of data analysis began to realize that there were many commonalities between what had been considered to be separate types of statistical models. For instance, Cohen (1978) derived a proof that interactions and partialed products were the same thing. Bentler (1976) used the LISREL formulation to illuminate similarities and differences between principal components analysis and factor analysis as he presented a model for what he called structural factor analysis. Bentler (1979) expanded on the theme to show how several kinds of models were all special cases of the LISREL formulation. McDonald proposed a set of principals that could be "... applied to give a unified treatment of a wide range of models for multivariate data, including models that have not yet been proposed." (McDonald, 1979, p. 22). Fraser and McDonald (1988) developed this idea as the COSAN software, which was later incorporated into SAS.

The Breakthrough: RAM Algebra and RAM Path Analysis

Here our history has come to the time where McArdle introduced the RAM specification. Consider the milieu of the late 70's. While room-sized mainframe computers are the way that LISREL models are fit, there is a new microcomputer that has just been released and that seems to be promising: the Apple II. Maybe some day people will be able to fit SEM models right on their own desks rather than waiting overnight for green-bar printouts from the data center. An easy, general way of specifying structural models might revolutionize how we go about converting our theories into testable hypotheses.

In 1978, as a postdoc at the University of Denver, McArdle first presented his ideas about how one might go about organizing theories drawn with path diagrams into a set of matrices that could then be optimized with respect to observed data (McArdle, 1978). This was followed a few months later by a presentation at the American Psychological Association's annual meeting where he called his method the Reticular Analysis Model (McArdle, 1979). The basic idea is that Wright's path diagrams provided a representation of the structure of a covariance matrix. The diagrams had four kinds of elements: manifest variables, latent variables, single headed arrows and double headed arrows. LISREL matrices make a distinction between manifest variables and latent variables that result in a particular structure of relations. For instance, in which matrix a regression coefficient is placed depends on whether it is predicting a manifest variable or a latent variable.

McArdle worked out that if one were to put regression coefficients for both manifest and latent variables into a single matrix, $\mathbf{A}$, one could radically reduce the total number of required matrices from LISREL's 8 or 10 matrices down to just 3. Similar logic applied to the covariance coefficients: A single matrix, $\mathbf{S}$, for covariances of all the variables could suffice. The well-known formula for the covariance of linear combinations could then be used to calculate the total model-implied covariance, $\mathbf{R}_{total}$, of the all the variables (manifest and latent).

$$\mathbf{R}_{total} = (\mathbf{I} + \mathbf{A})\mathbf{S}(\mathbf{I} + \mathbf{A})' \tag{1}$$

But this formula doesn't always work. Suppose we have three standardized variables, x_1, x_2, and x_3. Suppose further that

$$x_3 = b_2 x_2 + e_3 \tag{2}$$
$$x_2 = b_1 x_1 + e_2 \ . \tag{3}$$

There are five total variables in this system of equations: x_1, x_2, and x_3 are manifest, while e_2, and e_3 are model-implied, i.e., latent. If we set the rows and columns to be in the order $\{x_1, x_2, x_3, e_2, e_3\}$ then we can set $\mathbf{A}$ to

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ b_1 & 0 & 0 & 1 & 0 \\ 0 & b_2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

Note that the position of b_1 in $\mathbf{A}[2, 1]$ represents that the regression coefficient b_1 is predicting x_2 from x_1. Similarly, the position of b_2 in $\mathbf{A}[3, 2]$ represents that the regression coefficient b_2 is predicting x_3 from x_2. Also note that the latent variables e_2 and e_3 have an implied coefficient of 1.0 in Equations 2 and 3, so 1.0 is placed into the appropriate cells of $\mathbf{A}$.

In the same way, we can set up the variances and covariances that are to be estimated in the matrix $\mathbf{S}$, where

$$\mathbf{S} = \begin{bmatrix} 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & var(e_2) & 0 \\ 0 & 0 & 0 & 0 & var(e_2) \end{bmatrix} \tag{5}$$

Note that the variances of x_1, e_2, and e_3 are along the diagonal ($var(x_1) = 1.0$ since it is standardized). The variances of x_2, and x_3 are set to zero since their variances are completely accounted for by Equations 2 and 3.

However, now we find that Equation 1 does not follow Wright's path analysis rule that "The correlation between two variables can be shown to equal the sum of the products of the chains of path coefficients along all of the paths by which they are connected." (Wright, 1920, p 330). This can be easily seen by substituting Equation 3 into Equation 2. In order to calculate the products of the regression chain ($b_2 b_3$), we can calculate $\mathbf{A}^2$ and so now the sum of the products of the regression chain becomes

$$\mathbf{A} + \mathbf{A}^2 \tag{6}$$

and so

$$\mathbf{R}_{total} = (\mathbf{I} + \mathbf{A} + \mathbf{A}^2)\mathbf{S}(\mathbf{I} + \mathbf{A} + \mathbf{A}^2)' \tag{7}$$

Given that $\mathbf{I} = \mathbf{A}^0$, for each chain of regression coefficients of length n, we need to find

$$\mathbf{A}^0 + \mathbf{A}^1 + \mathbf{A}^2 + \ldots + \mathbf{A}^n \ , \tag{8}$$

a sum of a continued product (McDonald, 1978).

But now suppose that our model has a cyclic relation by adding one more path.

$$x_3 = b_2 x_2 + e_3 \tag{9}$$
$$x_2 = b_1 x_1 + e_2 \tag{10}$$
$$x_1 = b_3 x_3 + e_1 \ . \tag{11}$$

By the same logic as above, the sum of the products of the regression chain now becomes an infinite sum of a continued product,

$$\mathbf{R}_{total} = (\mathbf{A}^0 + \mathbf{A}^1 + \mathbf{A}^2 + \mathbf{A}^3 + \ldots + \mathbf{A}^\infty)\mathbf{S}(\mathbf{A}^0 + \mathbf{A}^1 + \mathbf{A}^2 + \mathbf{A}^3 + \ldots + \mathbf{A}^\infty)' \tag{12}$$

which would seem to be much harder to calculate. However, as McArdle (1980; 1984) noted,

$$\begin{aligned}(\mathbf{I} - \mathbf{A})(\mathbf{A}^0 + \mathbf{A}^1 + \mathbf{A}^2 + \mathbf{A}^3 + \ldots + \mathbf{A}^\infty) &= \mathbf{I} + (\mathbf{A} - \mathbf{A}) + (\mathbf{A}^2 - \mathbf{A}^2) + (\mathbf{A}^3 - \mathbf{A}^3)\ldots \\ &= \mathbf{I}\end{aligned}$$

So therefore

$$(\mathbf{A}^0 + \mathbf{A}^1 + \mathbf{A}^2 + \mathbf{A}^3 + \ldots + \mathbf{A}^\infty) = (\mathbf{I} - \mathbf{A})^{-1}$$

which means that

$$\mathbf{R}_{total} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{S}((\mathbf{I} - \mathbf{A})^{-1})' \tag{13}$$

applies to sums of products of coefficient chains of any length. This simple formula for the model implied covariance for all the variables can be filtered to just the measured variables by constructing an appropriate *measured variables* $\times$ *total variables* matrix, **F**, with a single 1.0 in each row corresponding to that measured variable's position in the **A** and **S** matrices. This, finally, is the RAM model which calculates $\mathbf{R}_{exp}$, the model-expected covariance matrix

$$\mathbf{R}_{exp} = \mathbf{F}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{S}((\mathbf{I} - \mathbf{A})^{-1})'\mathbf{F}' \ . \tag{14}$$

McDonald was the first to propose the notion of a general, two-matrix solution to the expected covariances of an SEM model (McDonald, 1978, p.61). His model was not isomorphic to Wright's tracing rules, but it was influential in McArdle's proposal later that year (McArdle, 1978). Equation 14 appears in the discussion at the end of Bentler & Weeks (1979). However, while they recognize that "It appears more general than [6] (*n.b.* the foundational equation in their article), and because it allows essentially all possible measurement levels and multivariate regressions, it also includes Weeks' (1978) seemingly more complex model [19]. Such a conclusion ignores some complexities of the problem." (Bentler & Weeks, 1979, p. 181). At the time, Bentler and Weeks did not appear to completely understand the generality of the model, nor the implications for path analysis. However, in subsequent years, Equation 14 became the foundation of Bentler's EQS software.

One of McArdle's main contributions came in how he realized that the calculations performed in Equation 14 mapped isomorphically to the set of path analysis rules originally defined by Wright (1920, 1934). Thus, each covariance in the model-expected covariance matrix, $\mathbf{R}_{exp}$ could be decomposed into additive *components of covariance.* Each component of covariance is the outcome of one traced path or, alternatively, one additive part of the matrix equation calculating $\mathbf{R}_{exp}$. This one-to-one correspondence, i.e., isomorphism, allows a path diagram to be a specification for one and only one matrix equation and similarly the matrix equation specifies one and only one path diagram. For this isomorphism to exist, McArdle (McArdle, 1980) realized that there was a missing diagrammatic element in previous path diagrams: there was not a diagrammatic element for variance terms in the **S** matrix. In standardized models—where the variance is fixed at 1.0 for all latent and observer predictors—the omitted variance terms had been assumed. But many path diagrams drawn without variance terms are ambiguous, and this impedes replication of analyses. The development of the double headed arrow from a variable to itself as representing a non-zero element in the **S** matrix allowed for a isomorphic (one to one) relationship between diagrams and equations and led to development of automatic graphic user interfaces for SEM.

Some Personal History

The first RAM-style path diagrams appeared in print (Horn & McArdle, 1980; McArdle, 1979, 1980) around the time when I first met him. At the time I had just published *Graphtrix*, a software package for text and graphics printing on the Apple II (S. M. Boker, 1980). Jack called me up for technical support on how to use it to include path diagrams into his manuscripts. We were both located in Denver, and so we met for lunch. Thus began a set of wide ranging discussions on the elements of path diagrams, graph theory, and why the double headed arrow variance term was so important to tracing rules. In these early diagrams (1979, 1980), McArdle's innovative representation of variance terms was already present, however variances and covariances did not yet include arrows at the ends of the arcs. A second graphical innovation in RAM diagrams is the use of a triangle to represent a column of ones in the data so that models for means can be specified in the path diagram (McArdle, 1986; McArdle & Epstein, 1987).

As McArdle presented his ideas it became evident that he was driven to implement the most general network of relations, the reticular relations proposed by Cattell (1965). I argued that "reticular" was an unnecessarily obscure term—why didn't he just call it Network Analysis Modeling. In his usual light-hearted manner, McArdle replied that the acronym NAM had obvious negative associations for many Americans. But primarily, he wanted to honor Cattell's contribution in helping generalize latent structure away from the strict input-output causal implications that had previously dominated statistical modeling.

There is a subtle point here. Input-output designs such as factor analysis, multiple regression, and mediation models all encourage the modeler to think in causal terms. On the other hand, a network model with feedback (reciprocal relations) is better framed in terms of bidirectional coupling or resonance. Strict causality becomes irrelevant in a highly connected feedback network—The answer to the causality question is always "yes" no matter which two variables and no matter in which order one picks. Modern network and dynamical systems models are recently beginning to be analyzed in terms of impulse response resonance, sidestepping the causality question entirely. In this way, McArdle was 30 years ahead of his time.

The **A** and **S** matrices have unique placement for each possible regression coefficient and each variance/covariance relation. McArdle said that one could think of the matrices as starting with a zero in every cell and add each regression coefficient or variance/covariance from a theory into the matrices. However, he said, one could also consider the matrices as starting as entirely filled and then zero out the regression coefficients and variance/covariances that *should not* appear in the diagram. The point here is a result from cognitive psychology: we tend to see what *is* present rather than what *is not* present. Structural equation modeling involves the balance between what is and what is not present in a path diagram. By focusing on what is not in the diagram, one focuses on the constraints that provide an opportunity for misfit. McArdle said this was akin to Sir Arthur Conan Doyle's Sherlock Holmes short story, *Silver Blaze*:

> Gregory (Scotland Yard detective): "Is there any other point to which you would wish to draw my attention?"
> Holmes: "To the curious incident of the dog in the night-time."
> Gregory: "The dog did nothing in the night-time."
> Holmes: "That was the curious incident." —Doyle, A.C. (1894, p. 22)

McArdle's point is well taken. The single and double headed arrows that are missing from a path diagram are exactly what makes a theory testable using goodness of fit statistics. The fact that many published path diagrams are artifactually missing critical elements that in fact existed in the calculation of a model's expectation has been a source of great consternation to McArdle, and he has crusaded to persuade SEM users to publish *complete* diagrams (e.g., Figure 4). The RAM-style path diagram is now a well established and often recommended standard (e.g., McDonald & Ho, 2002)

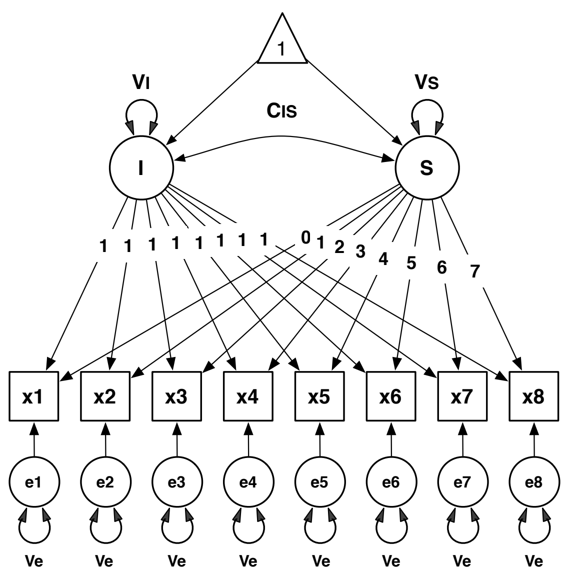

Figure 4. An intercept and slope latent growth curve model. Note the use of fixed basis functions to calculate the slope and intercept.

Out of these discussions, in 1982 I developed the RAMpath algorithm that allowed automatic computer-generated path diagrams from matrices (S. M. Boker, McArdle, & Neale, 2002). This algorithm was first implemented in order to automatically determine the longest regression chain so that the expected covariance matrix for a RAM-style model could be calculated without taking an inverse; thus enabling model-expected covariance matrices to be estimated on an early 48 kilobyte Apple II. Later, McArdle and I developed software that would automatically highlight each component of covariance one at a time on a path diagram (McArdle & Boker, 1990). This algorithm has since been used in software including Mx (M. C. Neale, Boker, Xie, & Maes, 1999), RAMONA (Browne & Mels, 1998), COSAN (Fraser & McDonald, 1988), OpenMx (S. Boker et al., 2009; M. Neale et al., 2016), Ωnyx (Oertzen, Brandmaier, & Tsang, 2015), and the R packages SEM (Fox, 2009) and RAMpath (Zhang, McArdle, Hamagami, & Grimm, 2012).

One early argument against the RAM formulation for use in model estimation is that its matrices are large and sparse. McArdle's argument was the Moore's law—the exponential increase in transistor density on CPUs—would make this efficiency penalty irrelevant. In his view, the bottleneck was not CPU time, but rather the time of the creative scientist. By making model specification as easy as possible, more creative data science would be accomplished. If one scientist spent more time attempting to shoehorn his theory into a cumbersome modeling framework whereas another scientist was able to translate her theory into a model more quickly and with fewer errors, then she would likely be the first to publish.

In the mean time, another advantage has become apparent. Due to the close link between graph theory and the model specification, techniques such as power equivalent models (Oertzen, 2010), RAMpart (Pritikin, Hunter, Oertzen, Brick, & Boker, 2017) and the RAMpath algorithm have allowed many common RAM-style models to automatically be more computationally efficient than the equivalent LISREL or MPlus specifications.

How Our Thinking About Modeling Has Changed

One important aspect of RAM is that it refocuses attention from the specifics of a particular modeling framework and brings to the foreground the psychological theory that is to be tested without few constraints on how that theory is instantiated in a model. This can be frustrating for those new to SEM, since there is a great deal of cognitive support provided by statistical techniques such as ANOVA and its regression alter-ego, the general linear model (GLM). New psychologists are trained to think in terms of predictors and outcomes. This is a useful technique for generalizing average effects to a population when combined with data from controlled experiments. However, the logic of ANOVA and GLM break down when conclusions are drawn from observational or quasiexperimental data. This can be illustrated in SEM by using the fact that ANOVA and GLM (and many other earlier techniques such as Factor Analysis, Principal Components, and Canonical Correlation) are special cases of RAM.

The "Reticular" in RAM emphasizes the fact that SEM needs to be understood as a being a network model. When Cattell (1965) proposed the idea of any observational model being embedded in a larger network of unobserved relations, few understood the wider implications. One of McArdle's main advances was to instantiate this idea of a general network model in a way that it could be estimated using cost function minimization optimizers (whether least squares, maximum likelihood, or Bayesian techniques such as Multi-Chain Monte Carlo). Methods for examining networks of variables have recently begun to evolve past RAM into techniques such as Exploratory Graph Analysis (EGA) (Golino & Epskamp, 2017), the GIMME algorithm (Gates, Molenaar, Hillary, Ram, & Rovine, 2010; Gates, Molenaar, Iyer, Nigg, & Fair, 2014), and SEM Trees (Brandmaier, 2011; Brandmaier, Oertzen, McArdle, & Lindenberger, 2013). These techniques are a testament to how far the field of SEM has progressed since the days when it was yet unclear how each separate type of statistical model related to one another. Even more impressive, our theories are beginning to break free of the trap of directed-effects causal thinking that has kept us from understanding the implications of resonant recurrent networks of relations—from appreciating Cattell's *reticule* as illustrated by Horn & McArdle (1980) in Figure 5.

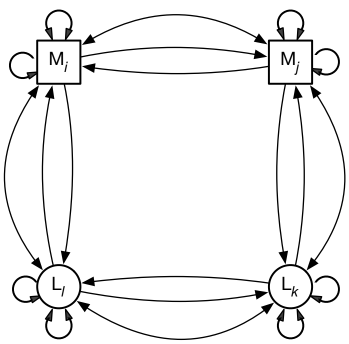

Figure 5. A reticule of variables (after Horn & McArdle, 1980, p. 194). Note that this diagram differs from Horn & McArdle in that arrows have been added to the covariance arcs and variances have been added to comply with McArdle's later full statement of RAM path diagrams. Single headed arrows from variables to themselves represent stability coefficients in an autoregressive sense. In the reticule it is useful to interpret single headed arrows as being effects occurring across time and double headed arrows as being contemporaneous. In this way, it the reticule can be seen as the basic building block of modern network algorithms.

Another important consequence of the RAM model and its isomorphic correspondence to path diagrams is that the way we discuss statistical models and teach SEM has been transformed. When students come into my office to ask questions about their theories and models, they overwhelmingly choose to bring a path diagram with them. In this way we can quickly communicate about the implications of models. After learning the tracing rules, students begin to actively explore the consequences of adding or subtracting regression relations in their models. This was made very clear to me when a second year graduate student came to see me after an SEM class period. She was excited and said she had just won her first argument with her advisor. She said she proved her point to her advisor using tracing rules and he was forced to agree. The didactic power of complete and isomorphic path diagrams is not to be underestimated.

The Future of RAM

The RAM model is not a static concept. It has been extended in several ways, for instance to account for multi-group and mixture distribution models. It has been used to fit state space and dynamical systems models such as Latent Differential Equations (LDE) (S. M. Boker, Neale, & Rausch, 2004). Recently, the OpenMx team has been exploring new extensions that have enabled multilevel models. The power of the RAM philosophy here can be seen in the efficiency of the RAMpart (Pritikin et al., 2017) model, an efficiency that derives from a graph theoretic transformation developed by von Oertzen (2010) where the portions of the RAM model that need to be inverted can be automatically isolated from the remainder of the multilevel model. Current work has been focusing on efficient and compact methods for estimating variances and covariances of products of variables. This will require an extension of the RAM matrices and new tracing rules. The RAM philosophy of *reticular action* of a network has been central to this work.

In short, the RAM model has been unreasonably influential in the development of the field of SEM over the past 40 years. I hope you will join me in thanking Jack McArdle for his incredibly helpful insights into the reticular structure of the network of relations between variables.

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